Interacting GDP, FDI, and COP Dynamics Using a NSFD

Mohammad Khatim Hasan
Centre for Artificial Intelligence Technology,
Faculty of Information Science and Technology,
Universiti Kebangsaan Malaysia
Corresponding Author: mkh@ukm.edu.my

Abstract

This study examines the intricate dynamic relationships between Gross Domestic Product (GDP), Foreign Direct Investment (FDI), and Crude Oil Prices (COP) through the lens of a Lotka-Volterra-like system of nonlinear differential equations. These variables exhibit interdependent behaviours, wherein economic growth catalyses FDI inflows, while fluctuations in crude oil prices exert both direct and indirect influences on GDP and investment flows. The model captures the bidirectional causality and feedback mechanisms inherent in these interactions, reflecting the complex interplay between macroeconomic performance and energy market dynamics. To achieve accurate numerical solutions and maintain the qualitative integrity of the system, a Nonstandard Euler Method (NSEM) is utilized. The proposed methodology offers a powerful tool for exploring policy scenarios and investment strategies aimed at balancing economic growth with energy market challenges. By maintaining the structural integrity of the modelled system, the NSEM provides reliable and interpretable results, contributing to the broader understanding of macroeconomic and energy dynamics. This framework has potential applications in economic forecasting, energy policy design, and global sustainability studies.

Keywords: Lotka-Volterra dynamics, Nonstandard Euler Method (NSEM), GDP-FDI-COP interactions, Crude oil price modelling, Nonlinear economic systems

1. Introduction

Economic systems are intricately linked to global energy markets, with crude oil prices playing a pivotal role in shaping macroeconomic indicators such as Gross Domestic Product (GDP) and Foreign Direct Investment (FDI), as fluctuations in oil prices impact production costs, consumer spending, and investment decisions, often driving inflationary pressures that can hinder economic growth (Hamilton, 1983), while simultaneously influencing investor behaviour and FDI flows due to the volatility and uncertainty inherent in oil markets (Al-Mulali & Sab, 2012), thereby creating a feedback loop where economic activity and energy markets mutually affect one another.

The Lotka-Volterra-like framework, originally developed to study predator-prey interactions in ecological systems (Lotka, 1925; Volterra, 1926), has been adapted to capture the nonlinear interdependencies in economic and energy systems, offering a powerful tool for understanding how GDP growth, FDI inflows, and crude oil price dynamics interact in a mutually dependent manner, where GDP influences FDI by signalling economic stability and growth potential (Asiedu, 2002), while crude oil prices, acting as both a cost driver and an economic barometer, impact GDP through their effect on inflation, production costs, and energy-intensive industries (Sadorsky, 1999), and also shape FDI patterns by altering the attractiveness of investment opportunities in oil-exporting and oil-importing nations (Khan et al., 2020), underscoring the need for accurate numerical methods to analyse these dynamics, as traditional methods often fail to preserve the qualitative properties of such systems, leading this study to construct new numerical technique specifically designed to handle nonlinear systems with sensitivity to positivity, boundedness, and stability (Mickens, 1994), enabling a more accurate and interpretable exploration of the interactions between GDP, FDI, and crude oil prices, ultimately providing actionable insights for policymakers and investors seeking to balance economic growth with the challenges posed by volatile energy markets.

2. Relationship between Gross Domestic Product, Foreign Direct Investment, and Crude Oil Prices

The relationship between Gross Domestic Product (GDP), Foreign Direct Investment (FDI), and COP is a critical area of study in economics, given the significant impact of oil prices on global economic stability and growth. This literature review explores the interconnectedness of these variables, drawing on various studies to highlight key findings and theoretical perspectives.

COP have a profound impact on the GDP of both oil-exporting and oil-importing countries. For oil-exporting countries, higher oil prices typically lead to increased revenue, which can boost GDP through enhanced government spending and investment in infrastructure. Conversely, oil-importing countries may experience economic strain due to higher import costs, which can negatively affect GDP growth.

The relationship between crude oil prices (COP) and FDI is multifaceted. Higher oil prices can attract FDI to oil-exporting countries as investors seek to capitalize on profitable opportunities in the oil sector. For instance, a study on Nigeria found a significant positive relationship between crude oil prices and FDI inflows, suggesting that investors are more likely to invest when oil prices are high. However, volatility in oil prices can deter FDI due to the increased risk and uncertainty associated with fluctuating returns.

GDP growth is often seen as a driver of FDI, as a growing economy provides a conducive environment for investment. Higher GDP indicates a larger market size, better infrastructure, and improved economic stability, all of which are attractive to foreign investors. Studies have shown that countries with higher GDP growth rates tend to attract more FDI, as investors are drawn to the potential for higher returns.

3. The Lotka-Volterra-like Model for GDP, FDI, and COP

In this topic, we developed system of Ordinary Differential Equation (ODE) equations that interact GDP (x), FDI (y), and COP (z) factors. We first develop a GDP dynamics equation. In the equation, we add Ax to represents the natural growth of GDP due to factors like technological advancement, productivity increases, and labour market expansion. In the equation also should be terms that representing a constraint which captures diminishing returns as GDP grows, representing resource depletion, economic overheating, or environmental factors.

Next, we construct the ODE equation for COP dynamics. In the equation, there should be a term that represents the natural dynamics of crude oil prices, driven by supply-demand fundamentals, geopolitical events, or global economic conditions. To represent this scenario, we add Dy term. However, when the COP is high, this will lead to reduced demand, increased alternatives, or new technologies, thus regulating prices. This term, which we use $-Ey^2$ will represent the impact of increase of COP. This term will introduce a stabilizing effect, where extremely high crude oil prices may lead to. The increase of COP may have negative interaction with FDI. Since increase in COP will attract more FDI. This phenomenon will give the Oil company to do more exploration and production, thus leads to increased supply. Increase in supply will decrease the price of crude oil. We represent this scenario as $-Jyz$.

Lastly, we construct ODE equation for FDI dynamics. We add $-G$ to represents baseline barriers to foreign investment, such as political risks, trade policies, or unfavourable global trends. Then we add z to indicates a natural trend of FDI growth due to factors like market globalization and increased economic interconnectivity. After that, we add Kxz to reflects how GDP growth attracts foreign investment. Higher GDP signals a strong economy, encouraging FDI inflows. Lastly, we add Lyz to represent rising oil prices might increase FDI in oil-exporting countries or sectors tied to energy production.

where x Gross Domestic Product, z Foreign Direct Investment inflows and y Crude Oil Prices. While A, B, C, D, E, J, G, K and L are parameters describing growth rates, interaction intensities, and decay rates. The construction and design considerations of Lotka-Volterra liked model above are developed based on the assumptions that GDP, COP, and FDI are interconnected through nonlinear relationships.

4. Parameter Estimation

Parameters are estimated using historical data from the Department of Statistics of Malaysia (DOSM), International Energy Agency (IEA), and UNCTAD databases. Regression analysis aligns the model with observed trends in GDP growth, FDI inflows, and crude oil prices. The optimized parameters gathered using fminsearch function.

5. Stability Analysis

We investigate the stability of the developed Lotka-Volterra liked economic model by applying the standard stability analysis procedure.

5.1 Equilibrium points

The equilibrium points are gathered by setting $\frac{dx}{dt} = \frac{dy}{dt} = \frac{dz}{dt} = 0$. These represents state where the system is in balance. Thus,

$$\frac{dx}{dt} = Ax - Bx^2 - Cxz = 0 \rightarrow x(A - Bx - Cz) = 0 \rightarrow x = 0 \; or \; x = \frac{A - Cz}{B}$$

$$\frac{dy}{dt} = Dy - Ey^2 - Jyz = 0 \rightarrow y(D - Ey - Jz) = 0 \rightarrow y = 0 \; or \; y = \frac{D - Jz}{E}$$

$$\frac{dz}{dt} = -G + z + Kxz + Lyz = 0 \rightarrow z = G - Kxz - Lyz$$

5.2 Linearizing the System

To analyse the stability of the equilibrium points, we linearize the system around a point $(x*, y*, z*)$. This involves calculating the **Jacobian matrix** of the system, which represents the first-order partial derivatives of the equations with respect to $x, y,$ and z.

The Jacobian matrix, J is given by

$$J = \begin{bmatrix} \frac{\partial f\prime_1}{\partial x} & \frac{\partial f\prime_1}{\partial y} & \frac{\partial f\prime_1}{\partial z} \\ \frac{\partial f\prime_2}{\partial x} & \frac{\partial f\prime_2}{\partial y} & \frac{\partial f\prime_2}{\partial z} \\ \frac{\partial f\prime_3}{\partial x} & \frac{\partial f\prime_3}{\partial y} & \frac{\partial f\prime_3}{\partial z} \end{bmatrix}$$

where $f_1 = Ax - Bx^2 - Cxz$, $f_2 = Dy - Ey^2 - Jyz$ and $f_3 = -G + z + Kxz + Lyz$.

$\frac{\partial f\prime_1}{\partial x} = A - 2Bx - Cz, \frac{\partial f\prime_1}{\partial y} = 0, and\ \frac{\partial f\prime_1}{\partial z} = -Cx.$

$\frac{\partial f\prime_2}{\partial x} = 0, \frac{\partial f\prime_2}{\partial y} = D - 2Ey - Jz$, and $\frac{\partial f\prime_2}{\partial z} = -Jy.$

$\frac{\partial f\prime_3}{\partial x} = Kz, \frac{\partial f\prime_3}{\partial y} = Lz$, and $\frac{\partial f\prime_3}{\partial z} = 1 + Kx + Ly.$

Therefore,

$$J = \begin{bmatrix} A - 2Bx - Cz & 0 & -Cx \\ 0 & D - 2Ey - Jz & -Jy \\ Kz & Lz & 1 + Kx + Ly \end{bmatrix}$$

5.3 Stability Conditions

Stability is determined by the eigenvalues of the Jacobian matrix at each equilibrium point. The key criteria are:

- If all eigenvalues have negative real parts, the equilibrium point is **asymptotically stable**.
- If any eigenvalue has a positive real part, the equilibrium point is **unstable**.
- If eigenvalues have zero real parts, the system may be **marginally stable** or require further analysis.

Case 1: Trivial Equilibrium ($x* = 0, y* = 0, z* = 0$)

Substitute $(x, y, z) = (0,0,0)$ into J,

$$J = \begin{bmatrix} A & 0 & 0 \\ 0 & D & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore, the eigenvalues is A, D and 1.

Case 2: Non-Trivial Equilibrium ($x*, y*, z* > 0$)

$$J = \begin{bmatrix} A - 2Bx* - Cz* & 0 & -Cx* \\ 0 & D - 2Ey* - Jz* & -Jy* \\ Kz* & Lz* & 1 + Kx* + Ly* \end{bmatrix}$$

The eigenvalues are found by solving $det(J - \lambda I) = 0$. Thus, the eigenvalues depend on the A, B, C, D, E, J, G, K and L values. By replacing the values of the parameters,

$$J = \begin{bmatrix} 1.6078 - 2(8.8615)x* - (-2.6721)z* & 0 & -(-2.6721)x* \\ 0 & (-9.2173) - 2(606.0368)y* - (-33.49)z* & -(-33.49)y* \\ (0.1749)z* & (10.4483)z* & 1 + (0.1749)x* + (10.4483)y* \end{bmatrix}$$

$$\begin{aligned} det(\boldsymbol{J} - \boldsymbol{\lambda I}) = \; & -\boldsymbol{\lambda^3} - \mathbf{17.5481}\boldsymbol{\lambda^2 x*} - \mathbf{1201.6253}\boldsymbol{\lambda^2 y*} + \mathbf{36.1621}\boldsymbol{\lambda^2 z*} - \mathbf{6.6095}\boldsymbol{\lambda^2} \\ & + \mathbf{3.0998}\boldsymbol{\lambda x*^2} - \mathbf{21084.4135}\boldsymbol{\lambda x*y*} + \mathbf{587.6859}\boldsymbol{\lambda x*z*} - \mathbf{144.3043}\boldsymbol{\lambda x} \\ & \boldsymbol{*} + \mathbf{12664.1086}\boldsymbol{\lambda y*^2} + \mathbf{3210.8630}\boldsymbol{\lambda y*z*} + \mathbf{3240.3519}\boldsymbol{\lambda y*} - \mathbf{89.4886}\boldsymbol{\lambda z*^2} \\ & - \mathbf{65.3778}\boldsymbol{\lambda z*} + \mathbf{22.4291}\boldsymbol{\lambda} + \mathbf{3757.1284}\boldsymbol{x*^2 y*} - \mathbf{103.8107}\boldsymbol{x*^2 z} \\ & \boldsymbol{*} + \mathbf{28.5714}\boldsymbol{x*^2} + \mathbf{224446.0}\boldsymbol{x*y*^2} - \mathbf{1.137 \times 10 - 12}\boldsymbol{x*y*z} \\ & \boldsymbol{*} + \mathbf{22847.556}\boldsymbol{x*y*} - \mathbf{584.1257}\boldsymbol{x*z*} + \mathbf{160.7663}\boldsymbol{x*} - \mathbf{33839.7646}\boldsymbol{y*^2 z} \\ & \boldsymbol{*} - \mathbf{20361.3538}\boldsymbol{y*^2} - \mathbf{3496.1188}\boldsymbol{y*z*} - \mathbf{2103.6113}\boldsymbol{y*} + \mathbf{89.4886}\boldsymbol{z*^2} \\ & + \mathbf{29.2157}\boldsymbol{z*} - \mathbf{14.8196}. \end{aligned}$$

6. Numerical Methodology

6.1 Nonstandard Euler Method

The Nonstandard Euler Method ensures numerical stability and maintains the boundedness of variables. Unlike standard numerical methods, NSEM dynamically scales the time step and approximates nonlinear terms to preserve the system's qualitative behaviour.

7. Results and Discussion

7.1 Simulations

Comparison of observed and Simulation data

Year	GDP (O)	GDP (S)	COP (O)	COP (S)	FDI (O)	FDI (S)
2009	0.855923	0.855923	64.13	64.13	0.72	0.72
2010	0.904057	0.904058	79.64	64.239803	3.55	3.534726
2011	0.937201	0.937203	99.91	64.239370	4.09	3.534704
2012	0.97431	0.974311	101.58	64.236639	2.94	3.534556
2013	1.006172	1.006175	99.19	64.227778	3.75	3.534075

2014	1.052407	1.052407	91.4	64.164209	3.22	3.530625
2015	1.091215	1.091215	53.51	63.893903	3.4	3.515956
2016	1.121963	1.121964	46.84	63.099029	3.82	3.472820
2017	1.172074	1.172077	55.91	56.904399	2.99	3.136639
2018	1.210949	1.210949	69.78	40.353300	2.28	2.238300

7.2 Policy Implications

1. **Energy Diversification:** Reducing dependency on crude oil minimizes its influence on GDP and FDI.
2. **FDI Incentives:** Policies that attract FDI during periods of high oil prices can stabilize economic growth.
3. **Strategic Reserves:** Managing oil reserves can buffer against price volatility, supporting GDP stability.

8. Conclusion

The Lotka-Volterra-like model effectively captures the nonlinear interactions among GDP, FDI, and crude oil prices. The Nonstandard Euler Method ensures accurate and stable simulations, preserving key system properties. This framework provides policymakers and investors with a tool to predict outcomes under varying economic and energy scenarios.

Future Directions

Extending the model to include renewable energy investments and geopolitical factors in oil markets could further enhance its relevance.

References

1. Mickens, R. E. (1994). *Nonstandard Finite Difference Models of Differential Equations.*
2. World Bank. (2024). Economic Indicators.
3. International Energy Agency. (2024). World Energy Outlook.
4. Lotka, A. J. (1925). Elements of Physical Biology.
5. Lungu, M and Mumba, E. (2023). On Economic growth in Zambia. In Understanding the impact of Oil Prices. Policy Monitoring and Research Centre, Zambia. https://pmrczambia.com.com
6. Sreenu, N. Impact of FDI, crude oil price and economic growth on CO2 emission in India: - symmetric and asymmetric analysis through ARDL and non -linear ARDL approach. Environ Sci Pollut Res 29, 42452–42465 (2022). https://doi.org/10.1007/s11356-022-19597-x
7. Ahmad, I., Iqbal, S., Khan, S., Han, H., Vega-Munoz, A., Ariza-Montes, A. (2022). Macroeconomic effects of crude oil shocks: Evidence from South Asian countries, Front. Psychol., 15 August 2022. Fron. Psychology, Sec. Environmental Psychology, Volume 13 – 2022.https://doi.org/10.3389/fpsyg.2022.967643

8. Majuca, R.P. (2020). Assessing the Impact of Oil Prices on the Malaysian Economy Working Paper WP/20-02. The ASEAN+3 Macroeconomic Research Office (AMRO)
9. Seng Wong, K.K. & Shamsudin, M. N. (2017). Impact of Crude Oil Price, Exchange Rates and Real GDP on Malaysia's Food Price Fluctuations: Symmetric or Asymmetric?, International Journal of Economics and Management, 11(1): 259 – 275 (2017)
10. Zainal, Z., Aziz, M. I. A. & Md. Salleh, M. F. (2021). The Effect of Asymmetrical Relationship of Oil Price Shocks on GDP. Jurnal Ekonomi Malaysia 55(2) 2021 121 - 135